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GAUGE VECTOR MASSES FROM FLAT CONNECTIONS?

Antti J. Niemi *†

Department of Theoretical Physics, Uppsala University
P.O. Box 803, S-75108, Uppsala, Sweden ‡

and

Department of Physics, University of British Columbia
6224 Acricultural Road, Vancouver, B.C. V6T 1Z1, Canada

We suggest that four dimensional massive gauge vectors could be described by coupling ordinary Yang-Mills theory to a topological gauge theory. For this the coupling should excite a nontrivial degree of freedom from the topological theory, corresponding to the longitudinal polarization of a massive gauge vector. If the coupling can be selected so that further degrees of freedom are not excited, one may entirely avoid particles such as the Higgs. Here we discuss a simple example of this idea, obtained by coupling standard Yang-Mills theory to the topological gauge theory of flat connections. We propose that our example might describe a renormalizable theory of massive gauge vectors with no additional physical degrees of freedom.

‡ permanent address

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* E-mail: ANTTI.NIEMI@TEORFYS.UU.SE

The Higgs mechanism [1] is the cornerstone in our present understanding of mass generation. However, even though all other particles of the Standard Model have been observed there is still no experimental evidence that a Higgs particle exists. The only indications come from systematic theoretical constructions that have excluded a number of alternatives in an impressive manner [2]: Even though low dimensional examples of both gauge invariant vector mass and dynamical symmetry breaking exist, the Higgs mechanism remains the sole method for generating a renormalizable mass for four dimensional nonabelian gauge vectors.

In the present Letter we try to develop an alternative to the Higgs mechanism. We suggest that massive gauge vectors could be described by coupling ordinary Yang-Mills theory to a topological theory [3], [4]. Unlike Higgs, topological fields do not describe physical degrees of freedom. Their Hilbert space has only a limited number of states. Usually these states describe the cohomology classes of a nilpotent BRST operator that characterises properties of the underlying four-manifold. If such a theory is coupled to a conventional theory, the coupling generically breaks the topological invariance and nontrivial degrees of freedom are excited. In particular, it may happen that if a topological theory is coupled to an ordinary Yang-Mills theory, a mass scale is introduced and these degrees of freedom become the longitudinal polarization of a massive gauge vector. If no other degrees of freedom are excited we may then have a renormalizable description of massive gauge vectors with no Higgs.

Here we consider a simple example of this idea. We couple the standard four dimensional $SU_Q(N)$ Yang-Mills theory with gauge field Q^a_μ and curvature $G^a_{\mu\nu}$ (subscripts Q *etc.* refer to the fields Q_μ *etc.*) to the $SU_A(N)$ BF theory [5], [4], a topological gauge theory that describes flat connections A^a_μ with curvature $F^a_{\mu\nu} \approx 0$. The BF theory is particularly interesting, since it can be viewed as a four dimensional analog [4] of the Chern-Simons theory that provides a gauge invariant vector mass in three dimensions. Furthermore, as a quantum field theory the BF theory is *finite* [6]. Hence its proper coupling to ordinary Yang-Mills theory might yield a renormalizable quantum field theory.

In the limit where all couplings between the two theories vanish we have a $SU_Q(N) \times SU_A(N)$ gauge symmetry. In particular we have two Gauss law generators corresponding

to the fields Q_μ and A_μ respectively. In this limit the only physical degrees of freedom are the two transverse components of the Yang-Mills field Q_μ, since gauge invariance and the flatness condition eliminate all physical excitations from the A_μ field.

We introduce a coupling between the two fields which breaks the $SU_Q(N) \times SU_A(N)$ gauge symmetry into the diagonal $SU_{Q+A}(N) \in SU_Q(N) \times SU_A(N)$ symmetry. Consequently only one Gauss law generator remains, corresponding to the diagonal $SU_{Q+A}(N)$ gauge transformations. In the absence of a $SU_A(N)$ Gauss law constraint for the A_μ field, the flatness condition is insufficient to eliminate all of its physical excitations. The degree of freedom that corresponds to $SU_{Q-A}(N)$ gauge transformations survives. This means that we are left with three physical degrees of freedom corresponding to the two transverse modes of $Q_\mu + A_\mu$ and the gauge mode of $Q_\mu - A_\mu$. If the coupling between Q_μ and A_μ has been selected properly, these degrees of freedom become the three polarizations of a massive gauge vector. If this theory can be renormalized, we have an alternative to the Higgs mechanism.

The four dimensional flat connection theory describes a $SU_A(N)$ gauge field A^a_μ subject to the flatness condition

$$F^a_{\mu\nu} \ = \ \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc} A^b_\mu A^c_\nu \ \approx \ 0 \tag{1}$$

and gauge invariance,

$$\mathcal{G}^a \ = \ D^{ab}_{A\mu} E^b_\mu \ = \ \delta^{ab}\partial_\mu E^b_\mu + f^{acb} A^c_\mu E^b_\mu \ \approx \ 0 \tag{2}$$

Here we have introduced a four-dimensional (Lagrangian) conjugate variable E^a_μ to all four components of the gauge field A^a_μ, with the (four dimensional) Poisson bracket[1]

$$\{E^a_\mu(x), A^b_\nu(y)\} \ \sim \ - \frac{\delta}{\delta A^a_\mu(x)} A^b_\nu(y) \ = \ - \delta^{ab}_{\mu\nu}(x-y)$$

so that (2) indeed generates gauge transformations of A_μ in four dimensions. The constraints (1), (2) obey a first class Poisson bracket algebra

$$\{ \ \mathcal{G}^a(x), \mathcal{G}^b(y) \ \} \quad = \quad f^{abc}\delta(x-y)\mathcal{G}^c(x)$$

[1]We are in a Minkowski space, but for simplicity we do not make a difference between upper and lower Lorentz indices. For example $\delta_{\mu\nu}$ in this E, A Poisson bracket is the Lorentz-covariant $\delta^\mu{}_\nu$ or more precisely components of a symplectic matrix, a structure which is independent of the metric.

$$
\begin{aligned}
\{\mathcal{G}^a(x), F^b_{\mu\nu}(y)\ \} &= f^{abc}\delta(x-y)F^c_{\mu\nu}(x) \\
\{F^a_{\mu\nu}(x), F^b_{\rho\sigma}(y)\} &= 0
\end{aligned} \tag{3}
$$

However, since $F^a_{\mu\nu}$ satisfies the Bianchi identity

$$
D^{ab}_{A\mu}F^b_{\nu\rho} + D^{ab}_{A\nu}F^b_{\rho\mu} + D^{ab}_{A\rho}F^b_{\mu\nu} = 0 \tag{4}
$$

the constraints (1) are reducible. Furthermore, since we also have the commutator

$$
D^{ac}_{A\mu}D^{cb}_{A\nu} - D^{ac}_{A\nu}D^{cb}_{A\mu} = -f^{abc}F^c_{\mu\nu} \tag{5}
$$

we have a first degree reducible constrained system which is on-shell second degree reducible [7].

We shall couple A^a_μ to the standard Yang-Mills field Q^a_μ so that the coupling introduces a mass scale, and in particular breaks the $SU_Q(N) \times SU_A(N)$ symmetry down to the diagonal $SU_{Q+A}(N)$ symmetry which is generated by

$$
\mathcal{G}^a = D^{ab}_{Q\mu}P^b_\mu + D^{ab}_{A\mu}E^b_\mu \tag{6}
$$

Here P^a_μ is the (four dimensional) conjugate to Q^a_μ,

$$
\{P^a_\mu(x), Q^b_\nu(y)\} = -\delta^{ab}_{\mu\nu}(x-y)
$$

In order to couple Q_μ and A_μ in a proper manner we recall [2] that tree level unitarity imposes strong restrictions on renormalizable theories with massive gauge vectors: Even though Yang-Mills theory with a Proca mass is one loop renormalizable [8], the requirement that tree amplitudes must be unitary indicates that Higgs fields are almost unavoidable [2].

In the present case, we observe that since both Q_μ and A_μ transform as gauge vectors under (6) the linear combination

$$
\mathcal{A}^a_\mu = \frac{1}{2}(Q^a_\mu + A^a_\mu) \tag{7}
$$

also transforms as a gauge vector. But since the inhomogeneous terms in the gauge transformed Q_μ and A_μ coincide, the linear combination

$$
\Phi^a_\mu = Q^a_\mu - A^a_\mu \tag{8}
$$

transforms like a Higgs field. If $\mathcal{F}^a_{\mu\nu}$ denotes the curvature of the gauge field $\mathcal{A}_\mu$,

$$\mathcal{F}^a_{\mu\nu} \;=\; \partial_\mu \mathcal{A}^a_\nu - \partial_\nu \mathcal{A}^a_\mu + f^{abc}\mathcal{A}^b_\mu \mathcal{A}^c_\nu$$

the *no-go* theorem of [2] tells us that we should couple Q_μ and A_μ in the following manner,

$$S \;=\; \int Tr\mathcal{F}^2_{\mu\nu} + \frac{1}{2} Tr D_\mu \Phi_\nu (D_\mu \Phi_\nu)^\dagger + \frac{m^2}{2f^2} Tr({\Phi_\mu}^\dagger \Phi_\mu - f^2)^2 + \{\Omega, \Psi\} \tag{9}$$

Here the first term contains the standard Yang-Mills action for Q_μ, D_μ is the covariant derivative *w.r.t.* (7) and Ω is a nilpotent BRST operator that we shall describe shortly: It should commute with the action (9) and it should take into account both the gauge transformations (6) and the flatness condition (1). The functional Ψ is a gauge fermion that determines our gauge fixing.

The action (9) specifies our attempt to describe massive gauge vectors. Without the identification (8) it can be viewed as a standard renormalizable Yang-Mills-Higgs action for a $SU(N)$ gauge field $\mathcal{A}_\mu$ and four species of Higgs fields Φ_μ, except that we have assigned a negative metric to the Higgs field Φ_0. Since we try to take into account the results of [2] as closely as possible, we have included the $Tr\Phi^4$ self-interaction but excluded *e.g.* terms like $Tr(D_\mu \Phi_\mu)^2$ which are also power-counting renormalizable: The action (9) is the most general power-counting renormalizable action which is consistent with a twisted version of Lorentz transformations, with the components of Φ_μ transforming as scalars instead of as vectors. Notice in particular, that we have not (yet) included the BF-term.

We shall now consider the BRST operator in (9). From [4], [5], [9] we conclude that it can be represented as a sum of two nilpotent BRST operators, Ω_{YM} describing the conventional Yang-Mills gauge transformations and Ω_{BF} describing the flatness condition

$$\Omega \;=\; \Omega_{YM} \;+\; \Omega_{BF} \tag{10}$$

i.e. the Yang-Mills and flatness symmetries separate in the BRST operator.

The construction of Ω_{YM} is straightforward, and the operator Ω_{BF} has also been discussed extensively, see *e.g.* [4], [5], [9], [6]. Here we introduce a slight variant of the standard approach which is more convenient for the present purposes. Our construction

of (10) will be based on the general algorithm described in [7], except that we shall apply it in a Lagrangian context. This is quite appropriate since the constraint algebra (3)-(5) is manifestly covariant. Hence it is isomorphic to a *canonical* constraint algebra in a *five* dimensional Hamiltonian theory. In particular, the corresponding Hamiltonian BRST operator should coincide with our four dimensional Lagrangian BRST operator.

We first consider the BRST operator Ω_{YM} that describes the gauge transformations generated by (6). We define anticommuting ghosts η^a and $\mathcal{P}^a$ with (four dimensional) brackets

$$\{\eta^a(x), \mathcal{P}^b(y)\} \;=\; -\,\delta^{ab}(x-y)$$

We also define extra ghosts $\bar{\eta}^a$ and $\bar{\mathcal{P}}^a$ with brackets

$$\{\bar{\eta}^a(x), \bar{\mathcal{P}}^b(y)\} \;=\; -\,\delta^{ab}(x-y)$$

and bosonic variables π^a, λ^a with

$$\{\pi^a(x), \lambda^b(y)\} \;=\; -\,\delta^{ab}(x-y)$$

We then introduce the nilpotent

$$\Omega_{YM} \;=\; \Omega_{YM}^{min} \;+\; \Omega_{YM}^{gf} \;=\; \eta^a \mathcal{G}^a + \frac{1}{2} f^{abc} \eta^a \eta^b \mathcal{P}^c + \lambda^a \bar{\eta}^a \tag{11}$$

Here Ω_{YM}^{min} is defined by the first two terms and describes the algebra of (6), while Ω_{YM}^{gf} coincides with the last term and is necessary for gauge fixing.

We now momentarily ignore the A_μ field and consider the standard Yang-Mills action

$$S \;=\; \int \frac{1}{4} TrG^2 \;+\; \{\Omega_{YM}, \Psi\} \tag{12}$$

Since TrG^2 is gauge invariant, this action is BRST invariant and in particular the corresponding path integral is invariant under local variations of Ψ. Selecting

$$\Psi \;=\; \bar{\mathcal{P}}^a \left(R^a(Q) + \lambda^a\right)$$

we get

$$S \;=\; \frac{1}{4} trG^2 + \eta^a \{\mathcal{G}^a, R^b\} \bar{\mathcal{P}}^b - \lambda^2 - \lambda^a R^a$$

If we choose

$$R^a[Q] \;=\; \sqrt{\frac{2}{\xi}} \partial_\mu Q^a_\mu$$

and redefine $\eta \rightarrow \sqrt{\frac{\xi}{2}}\eta$ and $\lambda \rightarrow \sqrt{\frac{\xi}{2}}\lambda$ which has unit Jacobian in the path integral, we find by integrating over the auxiliary field λ^a the familiar Lagrangian of Yang-Mills theory in the covariant R_ξ-gauge. This confirms that our Lagrangian point of view works. In particular (11) with (6) is a BRST operator that describes our $SU_{Q+A}(N)$ gauge transformations.

We now proceed to the construction of the BRST operator that describes the flatness constraint $F^a_{\mu\nu} \approx 0$ together with the structure (4) and (5). Following [7] we introduce anticommuting antisymmetric ghosts $\psi^a_{\mu\nu}$ and $\mathcal{X}^a_{\mu\nu}$, commuting ghosts ϕ^a_μ, p^a_μ and anticommuting ghosts c^a, b^a. We impose the (four dimensional) Poisson brackets

$$\begin{aligned} \{\mathcal{X}^a_{\mu\nu}(x), \psi^b_{\rho\sigma}(y)\} &= -(\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho})\delta^{ab}(x-y) \\ \{p^a_\mu(x), \phi^b_\nu(y)\} &= -\delta^{ab}_{\mu\nu}(x-y) \\ \{b^a(x), c^b(y)\} &= -\delta^{ab}(x-y) \end{aligned}$$

and define

$$\begin{aligned} \Omega_{BF} &= \Omega^{min}_{BF} + \Omega^{gf}_{BF} \\ &= \psi^a_{\mu\nu}G^a_{\mu\nu} + \phi^a_\rho\epsilon_{\rho\sigma\mu\nu}D^{ab}_{Q\sigma}\mathcal{X}^b_{\mu\nu} + c^a D^{ab}_{Q\mu}p^b_\mu + \frac{1}{8}c^a f^{abc}\epsilon_{\rho\sigma\mu\nu}\mathcal{X}^b_{\rho\sigma}\mathcal{X}^c_{\mu\nu} + \Omega^{gf}_{BF} \end{aligned} \quad (13)$$

Here Ω^{min}_{BF} describes the algebraic structure of the flatness condition. The first term relates to the flatness constraint (1), the second term corresponds to the Bianchi identity (4) and the third term takes into account the additional relation (5). But since (5) is an on-shell condition, these three terms define an operator which is nilpotent only on-shell $F_{\mu\nu} \approx 0$. The fourth term then ensures that Ω^{min}_{BF} is *off-shell i.e.* identically nilpotent.

As in (11), the (nilpotent) operator Ω^{gf}_{BF} is necessary to fix the gauge symmetries corresponding to the flatness condition. Reducibility implies that besides gauge symmetries associated with the original flatness condition we also have additional gauge symmetries that correspond to the following ghost constraints

$$\begin{aligned} \{p^a_\mu, \Omega^{min}_{BF}\} &= \epsilon_{\mu\nu\rho\sigma}D^{ab}_{Q\nu}\mathcal{X}^b_{\rho\sigma} \approx 0 \\ \{b^a, \Omega^{min}_{BF}\} &= -D^{ab}_{Q\mu}p^b_\mu \approx 0 \end{aligned}$$

and we must define Ω^{gf}_{BF} so that it also accounts for these ghost constraints. This leads to the *ghosts-for-ghosts* construction [7], which in the case of BF theories has been discussed extensively [4], [5], [9], [6].

We shall not repeat this construction here. It is (unfortunately) quite elaborate, and will not be necessary in the following. For us it is sufficient to know, that the Yang-Mills symmetry and the symmetries associated with the flatness condition separate in the BRST operator [4], [5].

We now need to combine (13) with (11). For this we introduce the following representations of the $SU(N)$ gauge algebra,

$$
\begin{aligned}
U^a &= \frac{1}{2} f^{abc} \mathcal{X}^b_{\mu\nu} \psi^c_{\mu\nu} \\
V^a &= f^{abc} \phi^b_\mu p^c_\mu \\
W^a &= f^{abc} c^b b^c
\end{aligned}
\tag{14}
$$

and extend Ω^{min}_{YM} in (11) to

$$
\Omega^{min}_{YM} = \eta^a (\mathcal{G}^a + U^a + V^a + W^a) + \frac{1}{2} f^{abc} \eta^a \eta^b \mathcal{P}^c \tag{15}
$$

This operator is nilpotent and describes the gauge transformations of our ghost fields. Furthermore, since

$$
\{\Omega^{min}_{YM}, \Omega^{min}_{BF}\} = 0 \tag{16}
$$

ensuring that the Yang-Mills and flatness symmetries indeed separate, we conclude that

$$
\Omega = \Omega^{min}_{YM} + \Omega^{min}_{BF} + \Omega^{gf}_{YM} + \Omega^{gf}_{BF} \tag{17}
$$

is a nilpotent BRST operator that projects the flatness condition to the gauge invariant subspace. Notice in particular, that (17) leaves the action (9) invariant.

We shall now proceed to fix the symmetries in (9). As a consequence of the structure (17) we may proceed in steps, by first fixing the Yang-Mills symmetry and then the symmetries associated with the flatness condition.

For the Yang-Mills symmetry we select the following gauge fermion,

$$
\Psi_{YM} = \frac{1}{2} \bar{\mathcal{P}}^a (\frac{1}{4} \lambda^a + \alpha \cdot \partial_\mu A^a_\mu + \beta \cdot \partial_\mu Q^a_\mu) \tag{18}
$$

where α, β specify different gauge conditions, and standard arguments imply that the path integral is independent of these parameters. For the action we get

$$
S = \int Tr \mathcal{F}^2_{\mu\nu} + \frac{1}{2} Tr D_\mu \Phi_\nu (D_\mu \Phi_\nu)^\dagger + \frac{m^2}{2f^2} Tr ({\Phi_\mu}^\dagger \Phi_\mu - f^2)^2
$$

$$
\begin{aligned}
+ &\quad \{\Omega_{YM}, \Psi_{YM}\} \; + \; \{\Omega_{BF}, \Psi_{BF}\} \\
= &\quad \int tr\mathcal{F}_{\mu\nu}^2 + \frac{1}{2}TrD_\mu\Phi_\nu(D_\mu\Phi_\nu)^\dagger + \frac{m^2}{2f^2}Tr({\Phi_\mu}^\dagger\Phi_\mu - f^2)^2 \\
- &\quad \frac{\alpha+\beta}{2\alpha}\cdot\bar{\mathcal{P}}^a\partial_\mu D_{Q\mu}^{ab}\eta^b - \frac{1}{8\alpha^2}\lambda^2 - \frac{1}{2\alpha}\lambda^a(\alpha\partial_\mu A_\mu^a + \beta\partial_\mu Q_\mu^a) + \{\Omega_{BF}, \Psi_{BF}\} \qquad (19)
\end{aligned}
$$

where Ψ_{BF} is a gauge fermion that fixes the remaining flatness symmetries.

We now momentarily ignore the $\{\Omega_{BF}, \Psi_{BF}\}$ term and set $\alpha = \beta = \xi$. We then have the standard R_ξ-gauge Yang-Mills-Higgs action with four Higgs fields Φ_μ. In particular, by eliminating λ^a and denoting $(A_\mu^1, A_\mu^2) = (Q_\mu, A_\mu)$ we find that our gauge fields propagate according to

$$
\Delta_{\mu\nu}^{ij} \; = \; \frac{1}{2}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} (\eta_{\mu\nu} - (1-\frac{1}{\xi})\frac{k_\mu k_\nu}{k^2})\frac{1}{k^2} & 0 \\ 0 & \eta_{\mu\nu}\;\frac{1}{k^2-2m^2} \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \qquad (20)
$$

where we recognize the familiar Yang-Mills and Higgs propagators. For the $\bar{\mathcal{P}}$ η ghosts we get similarly

$$
D \; = \; \frac{2\alpha}{\alpha+\beta}\frac{1}{k^2} \;\; \xrightarrow{\alpha=\beta=\xi} \;\; \frac{1}{k^2} \qquad (21)
$$

At this point we could redefine $\Phi_\mu^a \to \Phi_\mu^a + v_\mu^a$ where v_μ^a is a constant. Selecting $v^2 = f^2$ and *e.g.* improving (18) to the 'tHooft gauge fixing condition we then have the conventional spontaneous symmetry breaking approach to massive gauge fields [1].

We now return to the action (19), where we may also introduce the shift $\Phi_\mu^a \to \Phi_\mu^a + v_\mu^a$. However, since this corresponds to $Q_\mu^a \to Q_\mu^a + \frac{1}{2}v_\mu^a$ and $A_\mu^a \to A_\mu^a + \frac{1}{2}v^a$, and since the last term $\{\Omega_{BF}, \Psi_{BF}\}$ in (19) depends only on A_μ^a, complications will arise. These complications may be harmless since they only appear in BRST commutators. However, here we prefer to avoid them and instead we proceed by selecting

$$
\Psi_{BF} \; = \; \Psi_{BF}^{min} \; + \; \Psi_{BF}^{gf} \; = \; \frac{\gamma}{4}\,\mathcal{X}_{\mu\nu}^a F_{\mu\nu}^a \; + \; \Psi_{BF}^{gf} \qquad (22)
$$

where γ is another parameter, and by standard arguments the path integral does not depend on it. The remaining gauge fermion Ψ_{BF}^{gf} denotes the ghosts-for-ghosts contributions. For our action (22) gives

$$
\begin{aligned}
S \;\; = &\quad \int tr\mathcal{F}_{\mu\nu}^2 + \frac{\gamma}{2}trF_{\mu\nu}^2 + \frac{1}{2}TrD_\mu\Phi_\nu(D_\mu\Phi_\nu)^\dagger + \frac{m^2}{2f^2}Tr({\Phi_\mu}^\dagger\Phi_\mu - f^2)^2 \\
- &\quad \frac{\alpha+\beta}{2\alpha}\cdot\bar{\mathcal{P}}^a\partial_\mu D_{Q\mu}^{ab}\eta^b + \frac{1}{2}(\alpha\partial_\mu A_\mu^a + \beta\partial_\mu Q_\mu^a)^2 + \{\Omega_{BF}^{gf}, \Psi_{BF}^{gf}\} \qquad (23)
\end{aligned}
$$

More generally (and maybe after we have extended (15) so that as in (14) it includes an operator which gauge transforms $B_{\mu\nu}$) we could also define

$$\Psi^{min}_{BF} \;=\; \mathcal{X}^a_{\mu\nu}(B^a_{\mu\nu} + \frac{\gamma}{4}F^a_{\mu\nu})$$

where $B^a_{\mu\nu}$ is the analog of the Lagrange multiplier λ^a in (11). This gauge fermion would introduce an explicit BF term in the action. However, here we prefer (22).

The last term in (23), $\{\Omega^{gf}_{BF}, \Psi^{gf}_{BF}\}$, denotes the ghosts-for-ghosts contributions that are necessary to fix all gauge symmetries which are associated with the flatness condition. This term has been analyzed extensively in the literature [4]-[6], [9]. It introduces couplings to the gauge field A_μ and is known to have a complicated structure. However, since the BF theory is *finite* [6], this term should only yield power-counting renormalizable couplings and renormalizable propagators. Here we are interested in divergences that could render (9) nonrenormalizable, and it is natural to assume that such divergences should primarily originate from the terms that couple A_μ and Q_μ. These terms have been explicitly displayed in (23), the explicit form of $\{\Omega^{gf}_{BF}, \Psi^{gf}_{BF}\}$ should not be relevant for the present purposes.

From (23) we get for the gauge field propagator

$$\triangle^{ij}_{\mu\nu} \;=\; (\eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2})\begin{pmatrix} A & B \\ B & C \end{pmatrix} \;+\; \frac{k_\mu k_\nu}{k^2}\begin{pmatrix} a & b \\ b & c \end{pmatrix} \tag{24}$$

where

$$\begin{aligned} A \;&=\; \frac{1}{k^2}\,\frac{(1+\gamma)k^2 - m^2}{(1+\gamma)k^2 - (2+\gamma)m^2} \\ B \;&=\; -\frac{1}{k^2}\,\frac{m^2}{(1+\gamma)k^2 - (2+\gamma)m^2} \\ C \;&=\; \frac{1}{k^2}\,\frac{k^2 - m^2}{(1+\gamma)k^2 - (2+\gamma)m^2} \end{aligned} \tag{25}$$

and

$$\begin{aligned} a \;&=\; \frac{2}{(\alpha+\beta)^2}\,\frac{1}{k^2}\,\frac{(1+\beta^2)k^2 - 2m^2}{k^2 - 2m^2} \\ b \;&=\; \frac{2}{(\alpha+\beta)^2}\,\frac{1}{k^2}\,\frac{(1-\alpha\beta)k^2 - 2m^2}{k^2 - 2m^2} \\ c \;&=\; \frac{2}{(\alpha+\beta)^2}\,\frac{1}{k^2}\,\frac{(1+\alpha^2)k^2 - 2m^2}{k^2 - 2m^2} \end{aligned} \tag{26}$$

We point out that (if $\gamma \neq -1$) for large momenta $\triangle^{ij}_{\mu\nu}$ behaves like k^{-2} as it should in a renormalizable theory. Furthermore, if we set $\gamma \to 0$ and $\alpha = \beta = \xi$ we get back to (20). In particular, for $\xi = 1$ we have an analog of Feynman gauge with the potentially troublesome $k_\mu k_\nu$ structures in the propagator disappearing. We suggest that this is a strong argument for renormalizability.

We now propose that the flatness condition eliminates the Higgs field, and (23) describes only a massive vector propagator with mass m^2. For this we first set $\alpha, \gamma \to \infty$ which yields

$$\triangle^{ij}_{\mu\nu} \;=\; \begin{pmatrix} (\eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2})\,\frac{1}{k^2-m^2} & 0 \\ 0 & 2\frac{k_\mu k_\nu}{k^2}\frac{1}{k^2-2m^2} \end{pmatrix} \tag{27}$$

In this gauge the (physical) Q_μ propagates like a mass m^2 gauge vector in the Landau gauge Yang-Mills-Higgs theory, while A_μ becomes a "gradient ghost". Note that for an abelian theory (27) suggests that our mass coincides with the abelian Proca mass.

In the standard Yang-Mills-Higgs theory BRST invariance ensures that the $k^2 = 0$ pole that appears in the Landau gauge massive vector propagator disappears. By analogy we then argue that this should also happen in the present case. Furthermore, we shall now argue that the $k^2 = 2m^2$ pole in (27) must also cancel, leaving us with the $k^2 = m^2$ pole only. For this we consider the $\beta, \gamma \to \infty$ limit of (24). In this limit only the (11) component of $\triangle^{ij}_{\mu\nu}$ survives,

$$\triangle^{ij}_{\mu\nu} \;=\; (\eta_{\mu\nu} + \frac{k_\mu k_\nu}{k^2 - 2m^2})\,\frac{1}{k^2 - m^2}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{28}$$

If we compare this with the massive vector propagator in the R_ξ-gauge Yang-Mills-Higgs theory

$$\triangle_{\mu\nu} \;=\; \left(\eta_{\mu\nu} - (1 - \frac{1}{\xi})\frac{k_\mu k_\nu}{k^2 - \frac{1}{\xi}m^2}\right)\frac{1}{k^2 - m^2} \tag{29}$$

we observe that (28) corresponds to the $\xi = \frac{1}{2}$ gauge. In analogy with standard Yang-Mills-Higgs theory we then argue that in our case the $k^2 = 2m^2$ pole must also disappear. However, for this we need a Ward-like identity that relates Q_μ and A_μ which is possible only if A_μ does not entirely decouple. Indeed, since the A_μ self-interactions are proportional to γ certain diagrams containing both A_μ and Q_μ must survive as $\gamma \to \infty$. These diagrams have external Q_μ lines, and are connected to internal A_μ lines by the

propagator

$$- \frac{1}{\gamma}(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}) \, \frac{1}{k^2} \, \frac{m^2}{k^2 - m^2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \; + \; \mathcal{O}(\frac{1}{\gamma^2}) \tag{30}$$

These internal A_μ's then propagate with

$$\frac{1}{\gamma}(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}) \, \frac{1}{k^2} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \; + \; \mathcal{O}(\frac{1}{\gamma^2}) \tag{31}$$

and in order to produce a nontrivial $\gamma \to \infty$ limit, the factors of γ^{-1} that originate from (30), (31) must be exactly balanced by the factors of γ that arise from the A_μ self-interactions according to (23). This means that for general γ the diagrams that contain A_μ's and contribute to the S-matrix must satisfy some Ward-like identities. For example, if we take a derivative of the quantum partition function *w.r.t.* γ we find the constraint (1) in the weak form $< F^a_{\mu\nu}(x) F^a_{\mu\nu}(x) >= 0$. In particular, the $\gamma \to \infty$ limit is "unitary" in the sense that in this limit we explicitly obtain $F^a_{\mu\nu}(x) = 0$ as a δ-function constraint in the path integral.

The previous discussion suggests, that (23) is a renormalizable action that describes only massive vector fields, with no additional physical particles: All couplings between Q_μ and A_μ are power-counting renormalizable and the propagator (24) has the renormalizable k^{-2} large momentum behavior. Furthermore, there is also an analog of Feynman gauge where the potentially troublesome $k_\mu k_\nu$ structures in the gauge vector propagators disappear. Consequently the gauge fixing term $\{\Omega^{gf}_{BF}, \Psi^{gf}_{BF}\}$ remains as the only potential source of nonrenormalizable divergences. This term describes the ghosts-for-ghosts for the flatness condition, and since the BF theory is *finite* [6] all interactions that emerge from it must be power-counting renormalizable and all propagators must also have the renormalizable k^{-2} behavior at large momenta. Thus we argue that our action (23) should indeed be renormalizable. However, since our arguments are at best suggestive, this needs to be confirmed either by an explicit diagrammatic analysis or by a general proof.

Finally we point out, that (9) is not the only possible coupling between the Yang-Mills and BF theories. However, it appears to be the simplest one for which all propagators behave like k^{-2} for large momenta. For example, if we only include the coupling

$$m^2 Tr \Phi^2 \sim m^2 Tr(A_\mu - Q_\mu)^2$$

the propagators do not vanish like k^{-2} at large momenta. In this sense our construction is consistent with the *no-go* theorem in [2]. Indeed, the first three terms in (9) specify the most general action which is invariant under a twisted version of Lorentz transformations where the components of Φ_μ transform as scalars instead of as vectors. In our final action (23) this twisted Lorentz invariance is broken, but only by BRST commutators. However, we also point out that there are power-counting renormalizable terms that break our twisted Lorentz transformations and are not BRST commutators, but can not be directly excluded by the arguments in [2]. One candidate is $Tr(D_\mu\Phi_\mu)^2$ and another candidate is $Tr\Phi_\mu\Phi_\mu D_\nu\Phi_\nu$. Notice that the latter contributes only to the interactions.

In conclusion, we have investigated if massive gauge vectors could be described by coupling a Yang-Mills theory to a topological gauge theory. We have argued that if the topological theory describes flat connections, we get a renormalizable theory of massive gauge vectors. In particular, it appears that besides the three polarizations of the massive gauge vector there are no other physical particles. It would be very interesting to verify that this conjecture is indeed correct. Unfortunately, diagrammatic techniques for the BF theory have not yet been developed so that an effective perturbative investigation would be possible. Effective diagrammatic techniques are also needed if we wish to investigate the phenomenological consequences of our proposal.

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